Artificial Intelligence

Logic & Logic Agents

Chapter 7 (& Some background)

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Watch this lecture and download the slides from

http://jarrar-courses.blogspot.com/2011/11/artificial-intelligence-fall-2011.html

This lecture

General background about Logic, needed in the proceeding lectures:

- History of Logic, and a quick review of logic.
- Propositional Logic
- Logic Agents
- Logical inference = *reasoning*

Lecture Keywords:

Logic, History of Logic, knowledge representation, Propositional Logic, Logic agents, Knowledge-Based Agent, Validity of arguments, Tarski's world, Tarski's Semantics, Wumpus World Game, Inference, Deduction, Reasoning, Entailment, Logical Implication, Soundness, Completeness، satisfiable, Unsatisfiable, Validity, Falsifiable, tautology.

المنطق، المنطق الشكلي، تاريخ المنطق، تمثيل المعرفة، الاستنتاج، الاستنباط، صحة الحجمل المنطقية، الحدود

What is logic?

- A logic allows the axiomatization of the domain information, and the drawing of conclusions from that information.
  - Syntax
  - Semantics
  - Logical inference = *reasoning*

Computer Science deals with logic as a tool or a language to represent knowledge (even it is not true), and reason about it, i.e., draw conclusions automatically (…reaching intelligence).

Philosophy is more concerned with the truth of axioms, in the real world

Some History of Logic

322 BC

1037

1198

1960s

Automated Reasoning

2001

Complete Reasoning

?

Scalable Reasoning!

Review Of Propositional Logic Reasoning
Validity of Arguments

Example from [1]

$p \lor (q \lor r)$

$\sim r$

$\therefore p \lor q$

Is it a valid argument?
Also called: Query

				premises		conclusion
p	q	r	$q \lor r$	$p \lor (q \lor r)$	$\sim r$	$p \lor q$
T	T	T	T	T	F	
T	T	F	T	T	T	T
T	F	T	T	T	F	
T	F	F	F	T	T	T
F	T	T	T	T	F	
F	T	F	T	T	T	T
F	F	T	T	T	F	
F	F	F	F	F	T	

critical rows

In each situation where the premises are both true, the conclusion is also true, so the argument form is valid.

Review Of Propositional Logic Reasoning
Validity of Arguments

Example from [1]

$$p \to q \vee \sim r$$
$$q \to p \wedge r$$
$$\therefore p \to r$$

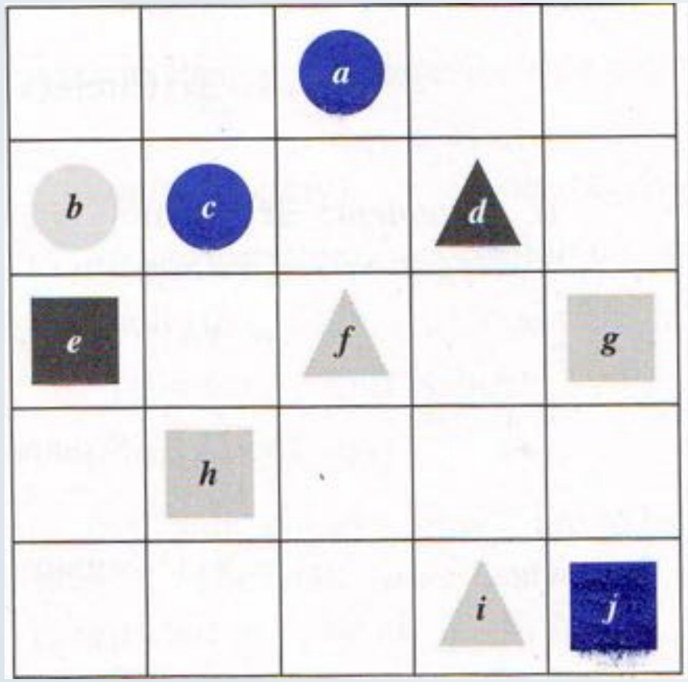
Is it a valid argument?
Also called: Query

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	F
T	F	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	T
F	T	F	T	T	F	T	F	T
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Quick Review: (First-Order-Logic)
Tarski's World example

Example from [1]

Describe Tarski's world using universal and external quantifiers



a. For all circles x, x is above f.

$$\forall x(\text{Circle}(x) \rightarrow \text{Above}(x, f)).$$

b. There is a square x such that x is black.

$$\exists x(\text{Square}(x) \wedge \text{Black}(x)).$$

c. For all circles x, there is a square y such that x and y have the same color.

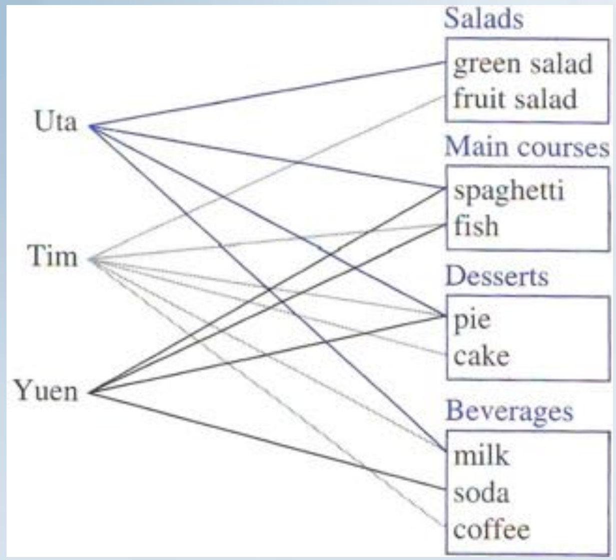
$$\forall x(\text{Circle}(x) \rightarrow \exists y(\text{Square}(y) \wedge \text{SameColor}(x, y))).$$

d. There is a square x such that for all triangles y, x is to right of y.

$$\exists x(\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \rightarrow \text{RightOf}(x, y))).$$

Quick Review: (First-Order-Logic)
Universal and Extensional Quantifiers Example

Example from [1]



a. $\exists$ an item I such that $\forall$ students S, S chose I.

b. $\exists$ a student S such that $\forall$ items I, S chose I.

c. $\exists$ a student S such that $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

d. $\forall$ students S and $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

There is an item that was chosen by every student. → true

There is a student who chose every available item. →false

There is a student who chose at least one item from every station. → true

Every student chose at least one item from every station → false.

Why Logic: Motivation Example

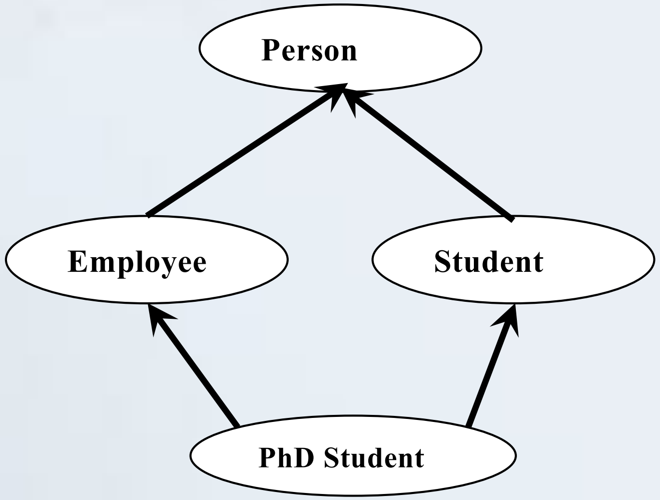
- Logic allows us to represent knowledge precisely (Syntax and Semantics).

$\forall$x Employee(x) → Person (x)

$\forall$x Student(x) → Person (x)

$\forall$x PhDStudent(x) → Student (x)

$\forall$x PhDStudent(x) → Employee (x)



- However, representation alone is not enough.

- We also need to process this knowledge and make use of it, i.e. Logical inference = (Reasoning).

Why Logic: Motivation Example

Reasoning:

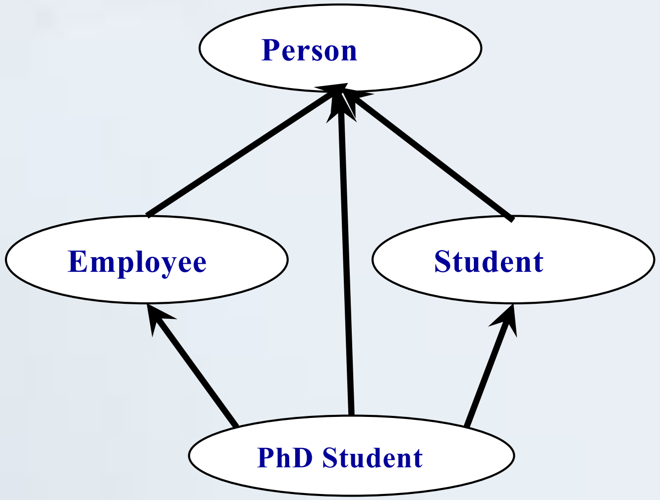
$\forall$x Employee(x) → Person (x)

$\forall$x Student(x) → Person (x)

$\forall$x PhDStudent(x) → Student (x)

$\forall$x PhDStudent(x) → Employee (x)

$\forall$x PhDStudent(x) → Person (x)



→How to process the above axioms to know that an axiom can be derived from another axiom.

Why Logic: Motivation Example

Reasoning:

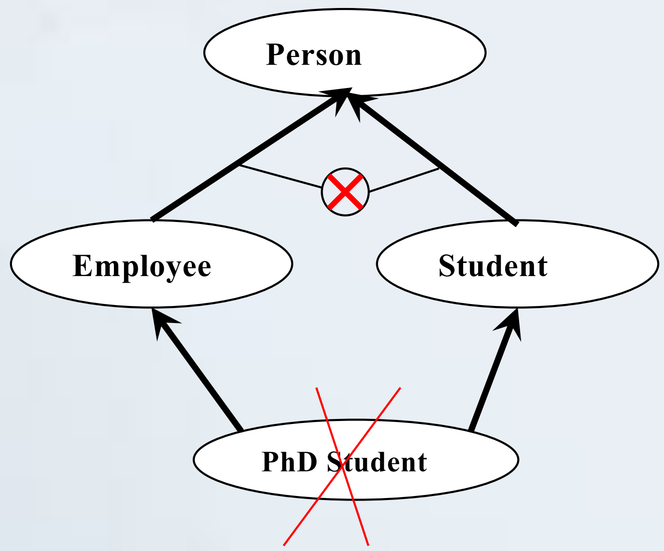
$\forall x$ Employee(x) → Person (x)

$\forall x$ Student(x) → Person (x)

$\forall x$ PhDStudent(x) → Student (x)

$\forall x$ PhDStudent(x) → Employee (x)

$\forall x$ Student(x) $\cap$ Employee (x) = $\emptyset$



➔ How to process the above axioms to know that an axiom can be derived from another axiom.

➔ Find contradictions (satisfiability)

➔ …etc.

Ch.7
Logic Agents

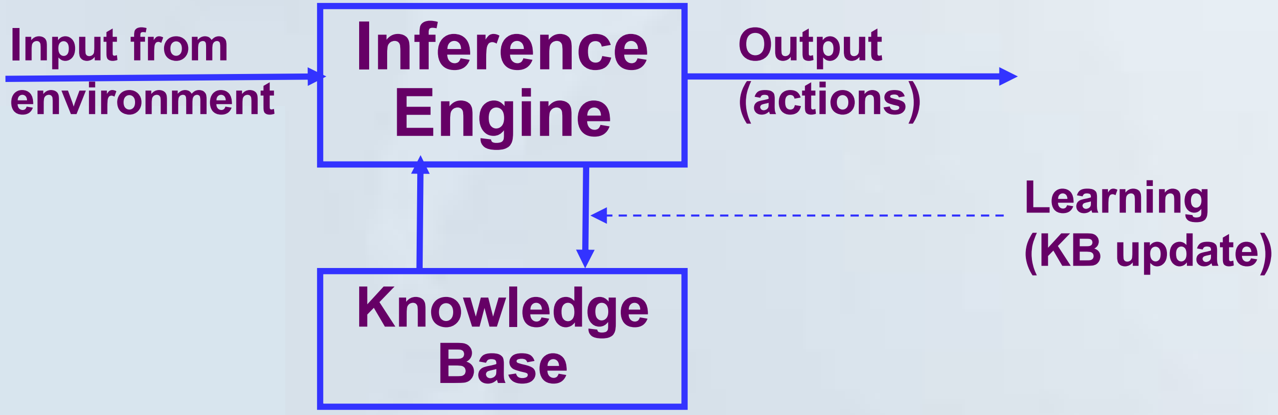
Most material is based on and improved from [2]

A Knowledge-Based Agent

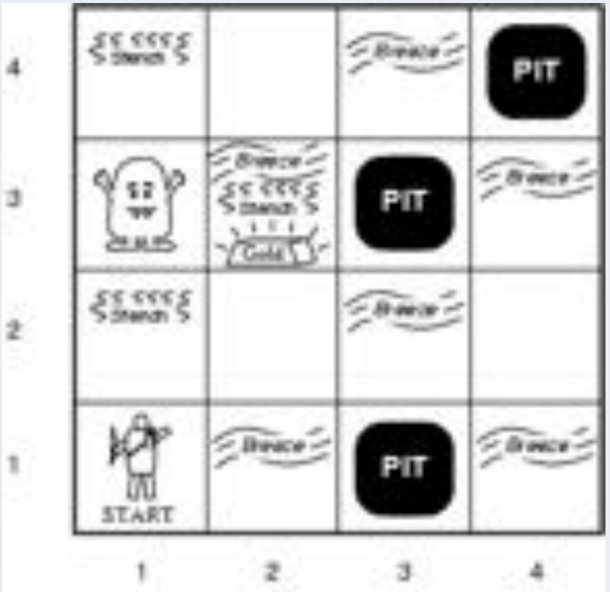
- A knowledge-based agent consists of a knowledge base (KB) and an inference engine (IE).
- A knowledge-base is a set of representations of what one knows about the world (objects and classes of objects, the fact about objects, relationships among objects, etc.)
- Each individual representation is called a sentence.
- The sentences are expressed in a knowledge representation language.
- Examples of sentences
  - The moon is made of green cheese
  - If A is true then B is true
  - A is false
  - All humans are mortal
  - Confucius is a human

A Knowledge-Based Agent

- The Inference engine derives new sentences from the input and KB
- The inference mechanism depends on representation in KB
- The agent operates as follows:
  - 1. It receives percepts from environment
  - 2. It computes what action it should perform (by IE and KB)
  - 3. It performs the chosen action (some actions are simply inserting inferred new facts into KB).



The Wumpus World



- Demo (Video)
- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square
- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Entailment –Logical Implication

- Entailment means that one thing follows from another:

$$KB \models \alpha$$

$$p \rightarrow q \vee \sim r$$
$$q \rightarrow p \wedge r$$
KB

$$? \therefore p \rightarrow r$$
α

- Knowledge base *KB* entails sentence α
  if and only if
  α is true in all worlds where *KB* is true

  - E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”

  - E.g., x+y = 4 entails 4 = x+y

  - Entailment is a relationship between sentences (i.e., Syntax) that is based on Semantics.

What is a "**Model**"? (in Logic)

- Logicians typically think in terms of models, which are formally *structured worlds* with respect to which truth can be evaluated.
- We say *m* is a model of a sentence α if α is true in *m*
- *M(α)* is the set of all models of α
- Then KB $\models$ α iff $M(KB) \subseteq M(\alpha)$

  E.g.

  KB = "Giants won" and "Reds won"

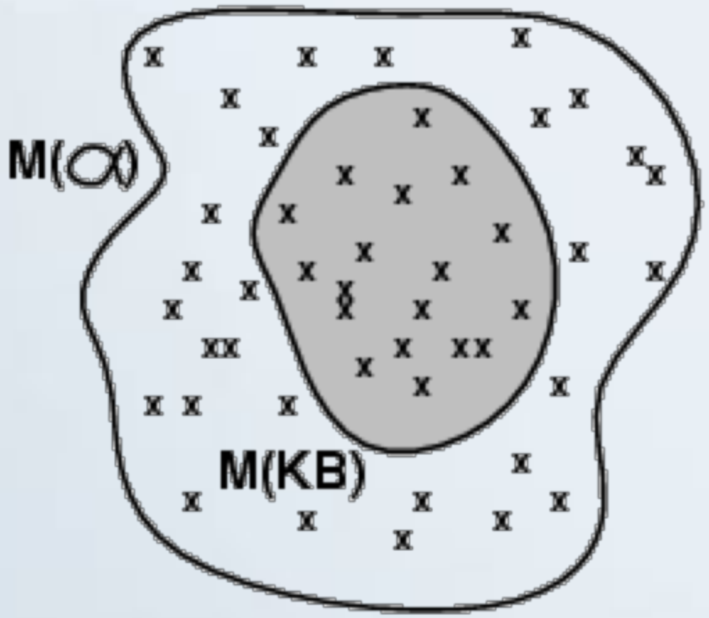
  α = "Giants won"

  Or

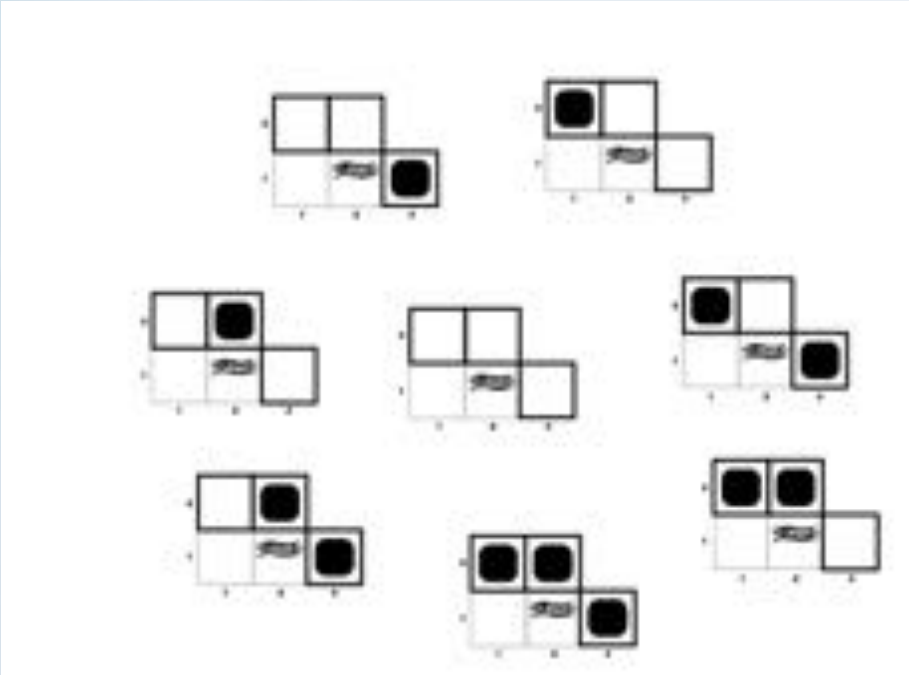
  α = "Red won"

  Or

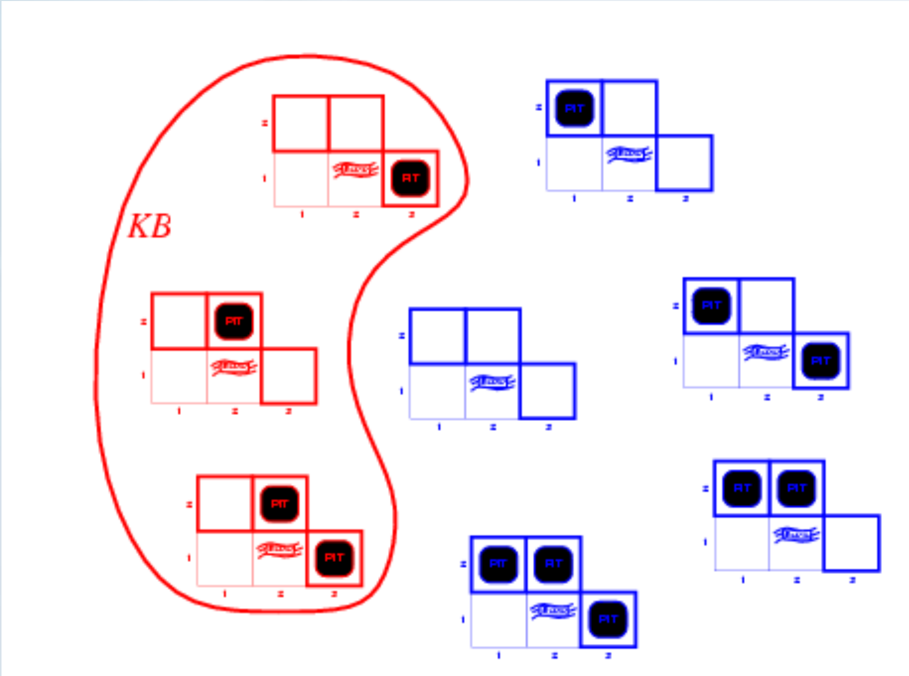
  α = either Giants or Red won



Wumpus Models

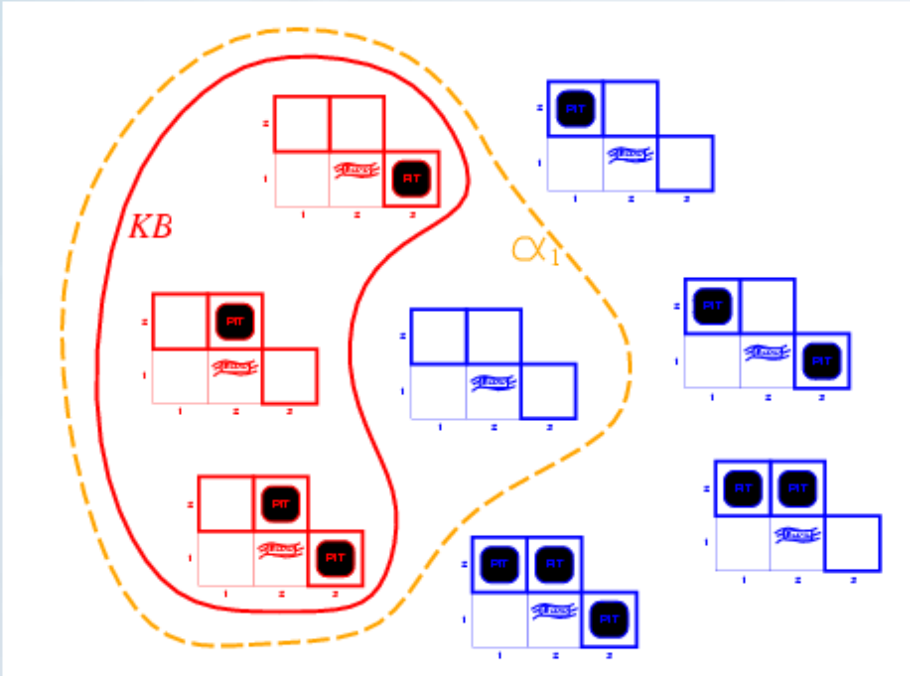


Wumpus Models



- *KB* = Wumpus-world rules + observations

Wumpus Models



- *KB* = wumpus-world rules + observations
- α_1 = "[1,2] is safe",

KB $\models \alpha_1$, proved by model checking

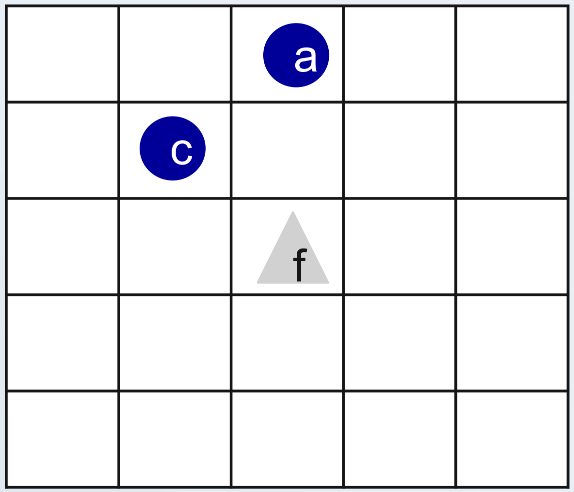
Another example

- Construct a model satisfying the following α

  $\alpha = \forall x\ (\text{Circle}(x) \rightarrow \text{Above}\ (x, f))$

- Model:

  Circle(*a*)
  Circle(*c*)
  Triangle(*f*)
  a in [3,5]
  c in [2,4]
  f in [3,3]



We say that this model is an interpretation for α

Properties of Inference Procedures

Inference = Deduction = Reasoning

$KB \vdash_i \alpha$

- $KB \vdash_i \alpha$ = sentence α can be derived from *KB* by **procedure** ***i***

- Soundness: *i* is sound if
  whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

- Completeness: *i* is complete if
  whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

- That is, the procedure will answer any question whose answer follows from what is known by the *KB*.

Propositional logic

Propositional logic: Syntax

- Propositional logic is the simplest logic –illustrates basic ideas

Countable alphabet Σ of **atomic propositions**: $a, b, c, \ldots$.

Propositional formulas:

ϕ, ψ	$\longrightarrow$	a	*atomic formula*
	$\mid$	$\bot$	*false*
	$\mid$	$\top$	*true*
	$\mid$	$\neg\phi$	*negation*
	$\mid$	$\phi \wedge \psi$	*conjunction*
	$\mid$	$\phi \vee \psi$	*disjunction*
	$\mid$	$\phi \rightarrow \psi$	*implication*
	$\mid$	$\phi \leftrightarrow \psi$	*equivalence*

- **Atom:** atomic formula
- **Literal:** (negated) atomic formula
- **Clause:** disjunction of literals

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 (= 2^3) possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{true} \vee \mathit{false}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$$

A simple knowledge base

Let $P_{i,j}$ be true if there is a pit in [i, j].
Let $B_{i,j}$ be true if there is a breeze in [i, j].

Knowledge Base:

$\neg P_{1,1}$ — There is no pit in [1, 1]

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$ — $B_{1,1}$ is breezy if and only if there is a pit in a neighboring square

$B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

➔ These sentences are true in all wumpus worlds.

$\neg B_{1,1}$

$B_{2,1}$

Validity and Satisfiability

A sentence is valid if it is true in all models,

e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

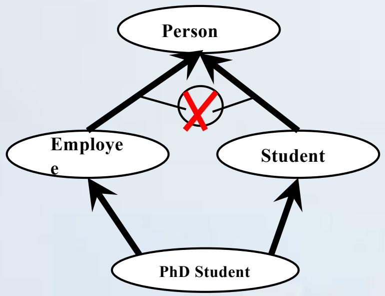
e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable



Validity and Satisfiability

An interpretation I is a **model** of α:

$$I \models \alpha$$

A formula α is

- **Satisfiable**, if there is some I that satisfies α ,
- **Unsatisfiable**, if α is not satisfiable,
- **Falsifiable**, if there is some I that does not satisfy α ,
- **Valid** (i.e., a tautology), if every I is a model of α .

Two formulas are logically equivalent ($\alpha \equiv \psi$), if for all I :

$$I \models \alpha \textbf{ iff } I \models \psi$$

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