Heuristic Informed Search Algorithms

(Chapter 3)

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Acknowledgement:
This lecture is based on (but not limited to) chapter 3 in "S. Russell and P. Norvig: *Artificial Intelligence: A Modern Approach"*.

Discussion and Motivation

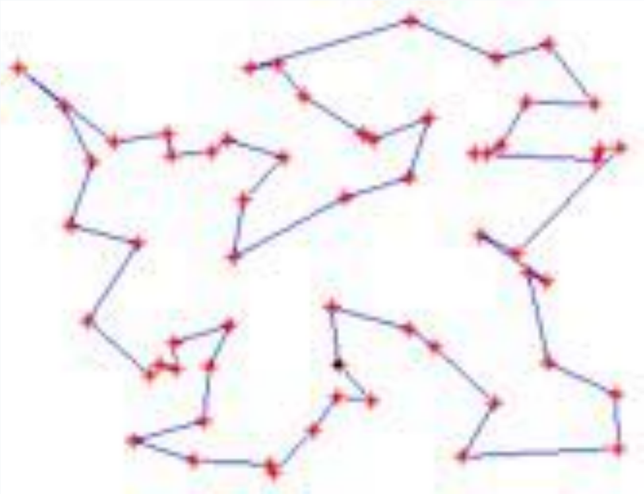
How to determine the minimum number of coins to give while making change?

➔ The coin of the highest value first ?

Discussion and Motivation

Travel Salesperson Problem

Given a list of cities and their pair wise distances, the task is to find a shortest possible tour that visits each city exactly once.



- Any idea how to improve this type of search?
- What type of information we may use to improve our search?
- Do you think this idea is useful: (At each stage visit the unvisited city nearest to the current city)?

Best-first search

Idea: use an evaluation function *f(n)* for each node

- family of search methods with various evaluation functions (estimate of "desirability“)
- usually gives an estimate of the distance to the goal
- often referred to as *heuristics* in this context

→Expand most desirable unexpanded node.

→A heuristic function ranks alternatives at each branching step based on the available information (heuristically) in order to make a decision about which branch to follow during a search.

<u>Implementation</u>:

Order the nodes in fringe in decreasing order of desirability.

Special cases:

- greedy best-first search
- A^* search

Romania with step costs in km

Suppose we can have this info (SLD)
➔ How can we use it to improve our search?

Greedy best-first search

- Greedy best-first search expands the node that appears to be closest to goal.
- Estimate of cost from *n* to *goal* ,e.g., $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest.

Utilizes a heuristic function as evaluation function

- $f(n) = h(n)$ = estimated cost from the current node to a goal.
- Heuristic functions are problem-specific.
- Often straight-line distance for route-finding and similar problems.
- Often better than depth-first, although worst-time complexities are equal or worse (space).

Greedy best-first search example

Example from [1]

Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
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Vaslui	199
Zerind	374

Greedy best-first search example

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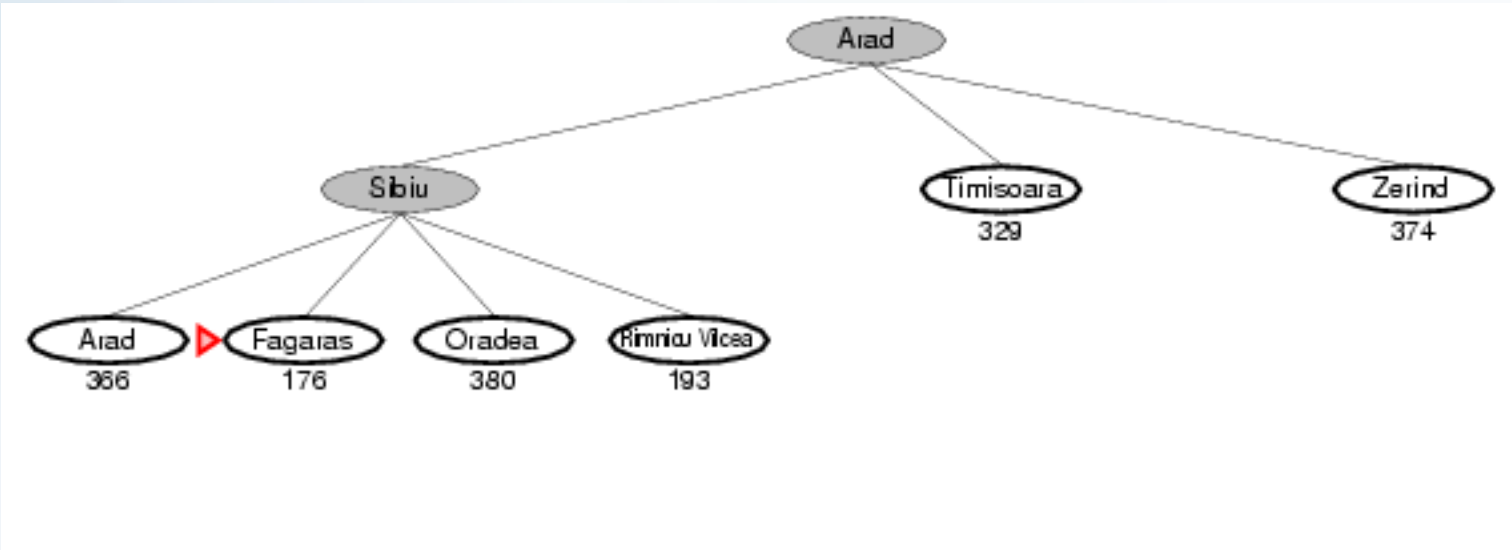
Greedy best-first search example



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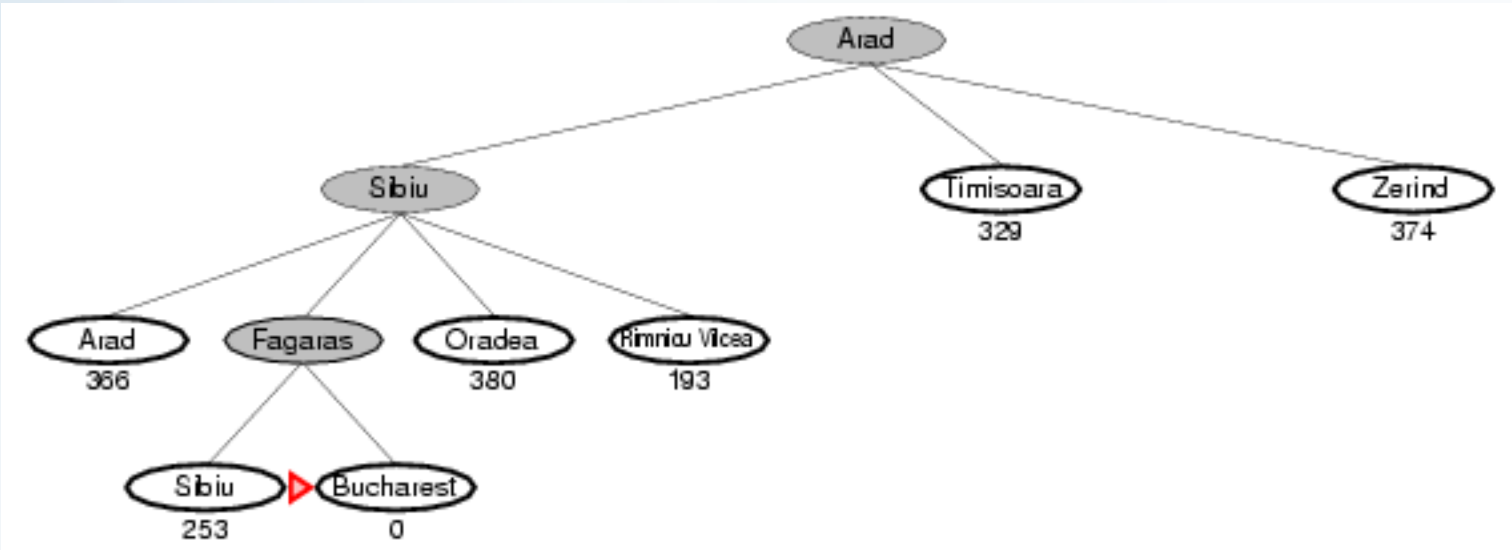

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Greedy best-first search

```
function GREEDY-BEST-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE : /* Cost f(n) = h(n) */

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

Properties of greedy best-first search

Complete: No – can get stuck in loops (e.g., Iasi → Neamt → Iasi → Neamt → ….)

Time: $O(b^m)$, but a good heuristic can give significant improvement

Space: $O(b^m)$ -- keeps all nodes in memory

Optimal: No

b	branching factor
m	maximum depth of the search tree

Discussion

Do you think $h_{SLD}(n)$ is admissible?

Would you use $h_{SLD}(n)$ in Palestine? How? Why?

Did you find the Greedy idea useful?

→ Ideas to improve it?

A* search

Idea: avoid expanding paths that are already expensive.

Evaluation function = path cost + estimated cost to the goal

$$f(n) = g(n) + h(n)$$

-$g(n)$ = cost so far to reach n

-$h(n)$ = estimated cost from n to goal

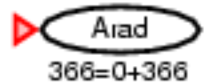
-$f(n)$ = estimated total cost of path through n to goal

Combines greedy and uniform-cost search to find the (estimated) cheapest path through the current node

- Heuristics must be admissible
  - Never overestimate the cost to reach the goal
- Very good search method, but with complexity problems

A* search example

Example from [1]



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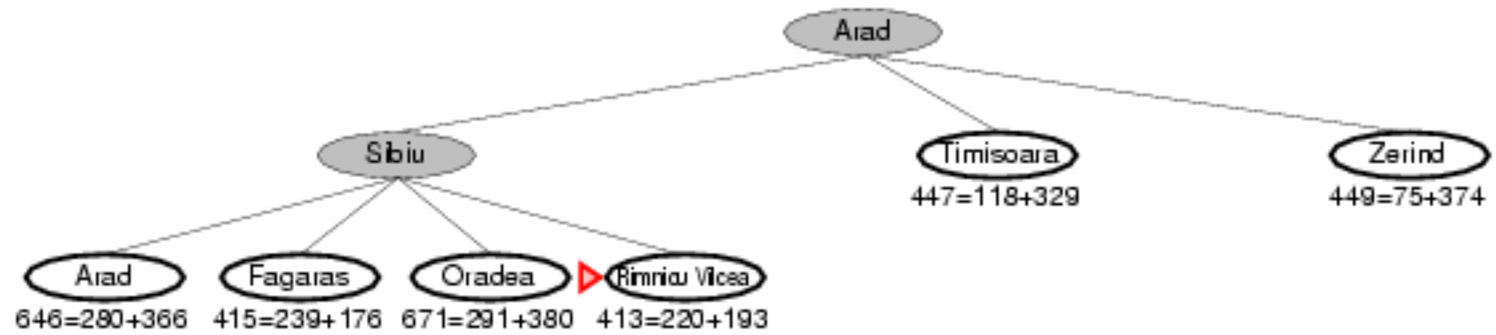
A* search example



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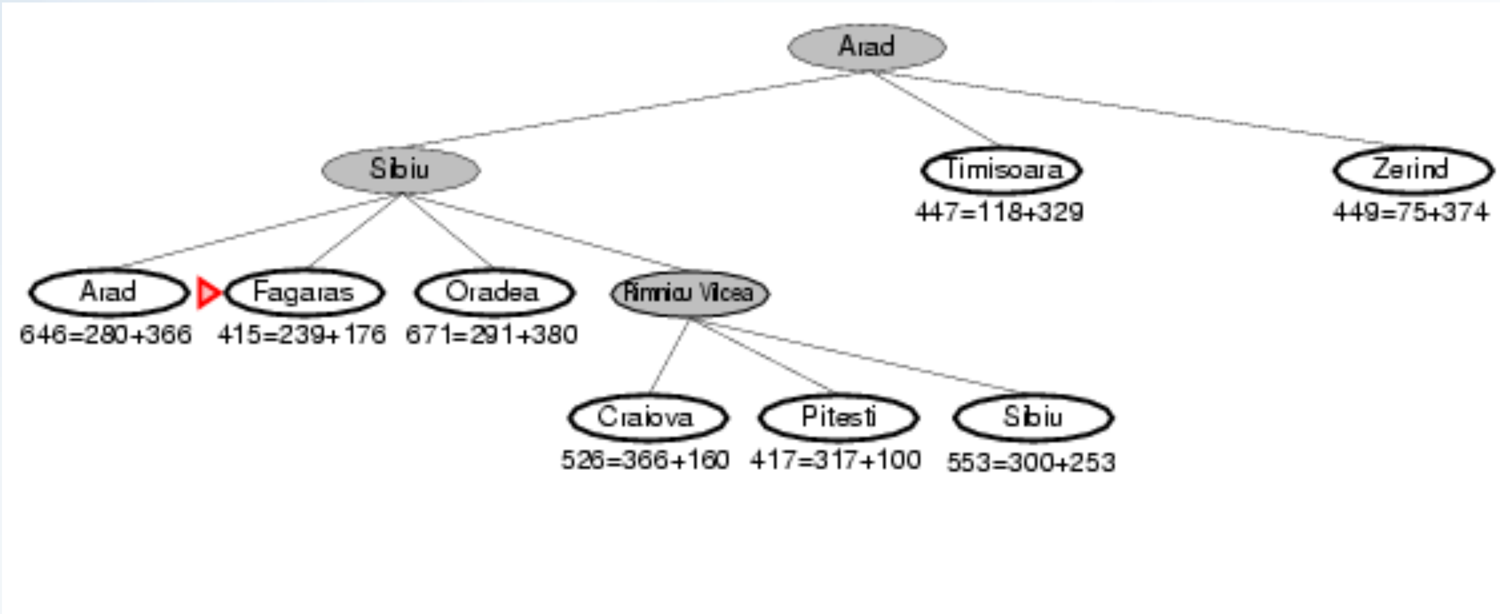

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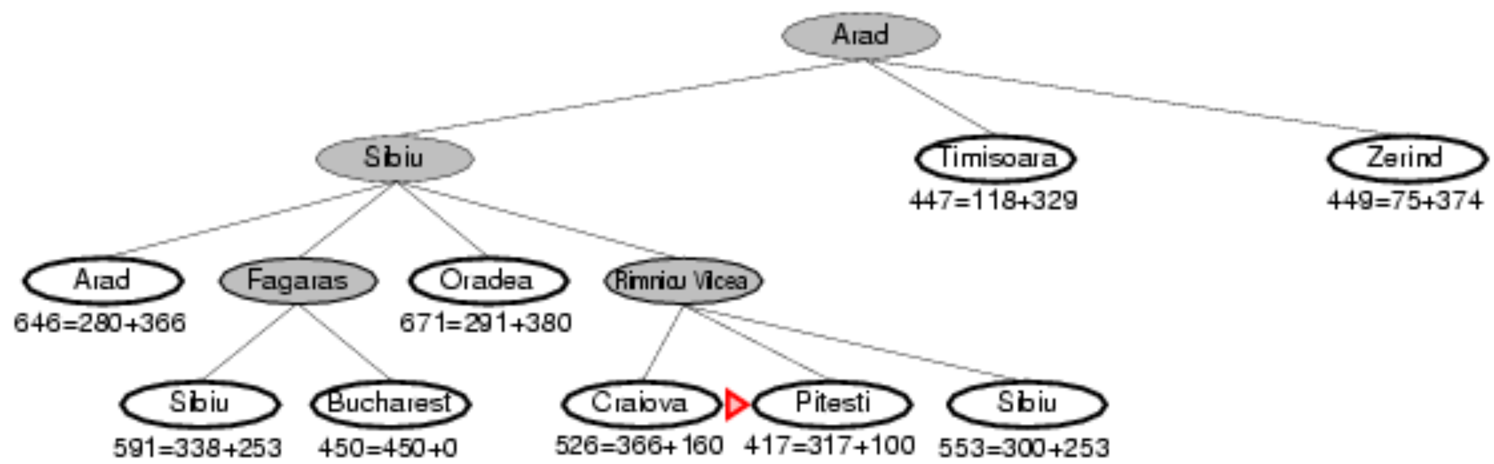

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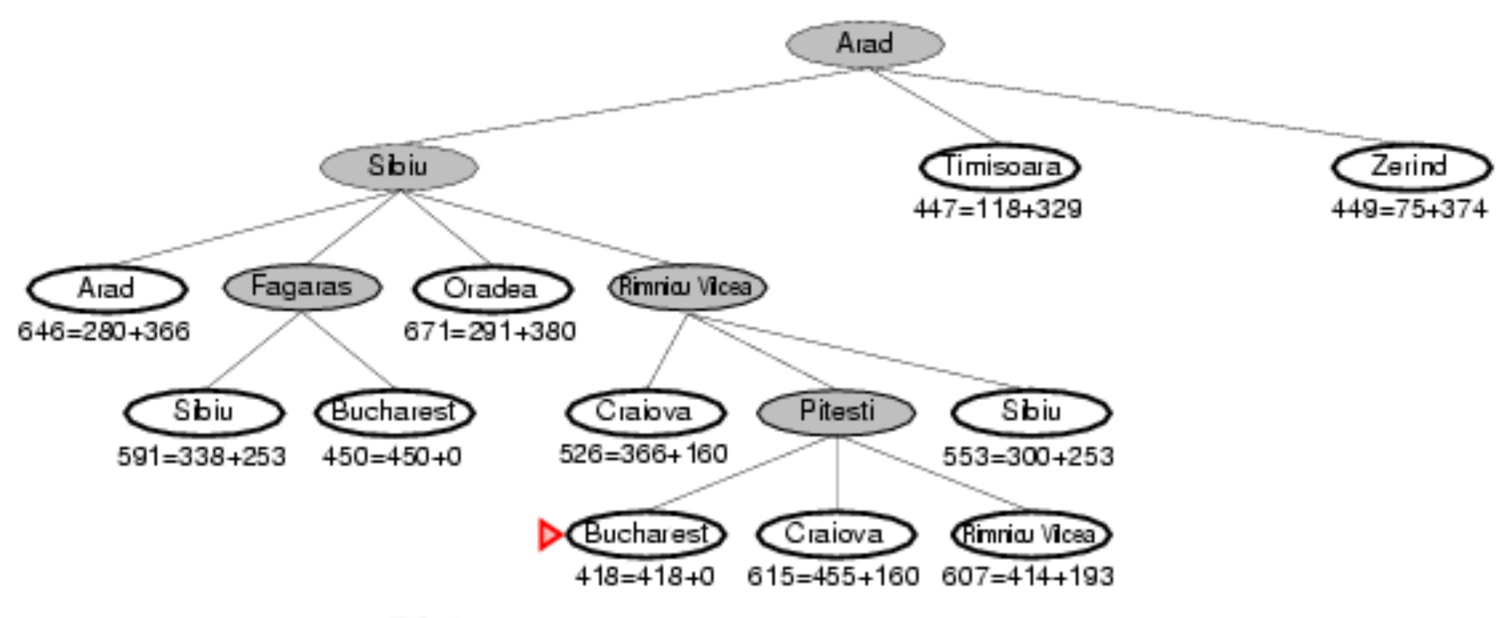

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A* Algorithm

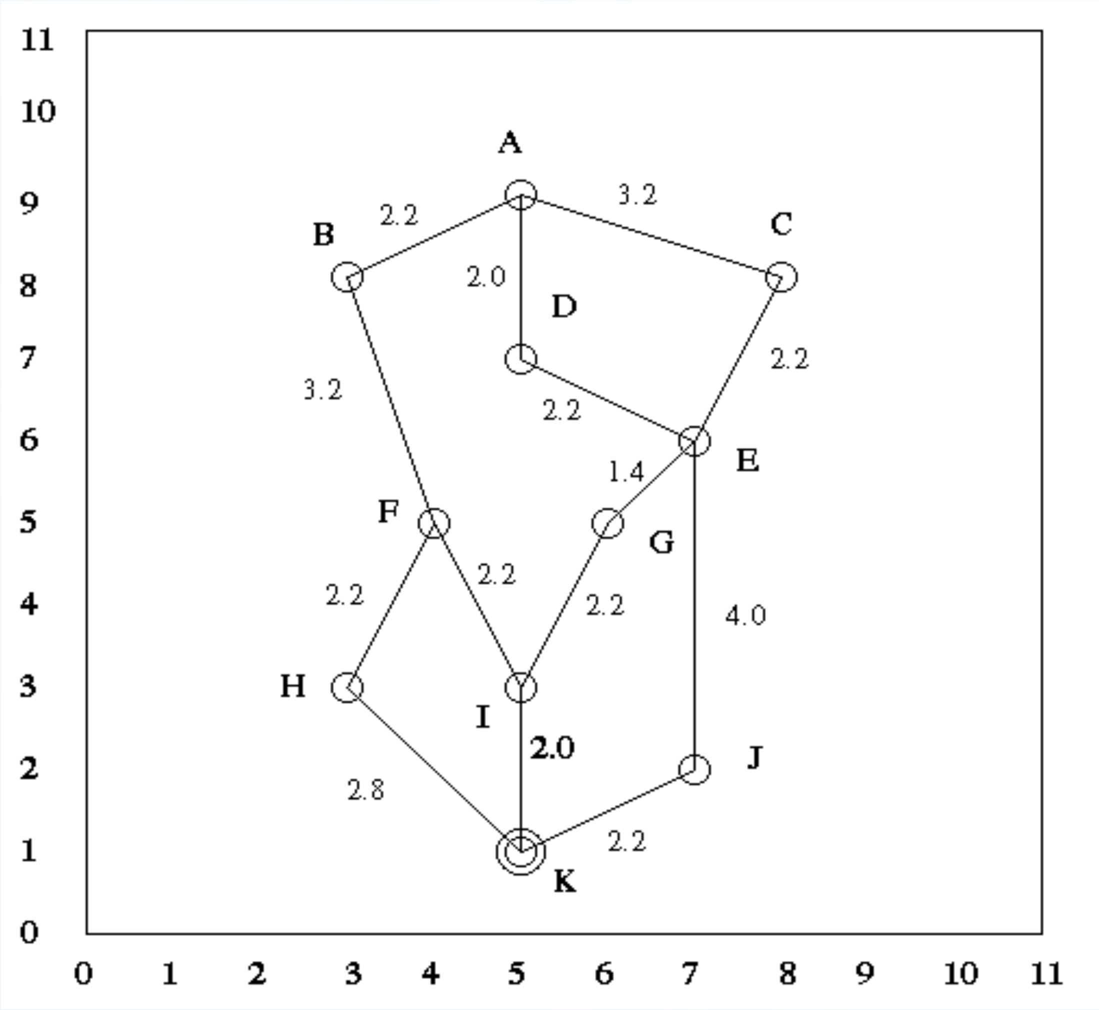
```
function A-STAR-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE : /* Cost f(n) = g(n) + h(n) */

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

A* Exercise

How will A* get from Iasi to Fagaras?

A* Exercise

Node	Coordinates	SL Distance
A	(5,9)	8.0
B	(3,8)	7.3
C	(8,8)	7.6
D	(5,7)	6.0
E	(7,6)	5.4
F	(4,5)	4.1
G	(6,5)	4.1
H	(3,3)	2.8
I	(5,3)	2.0
J	(7,2)	2.2
K	(5,1)	0.0



Solution to A* Exercise

Node	Coordinates	SL Distance
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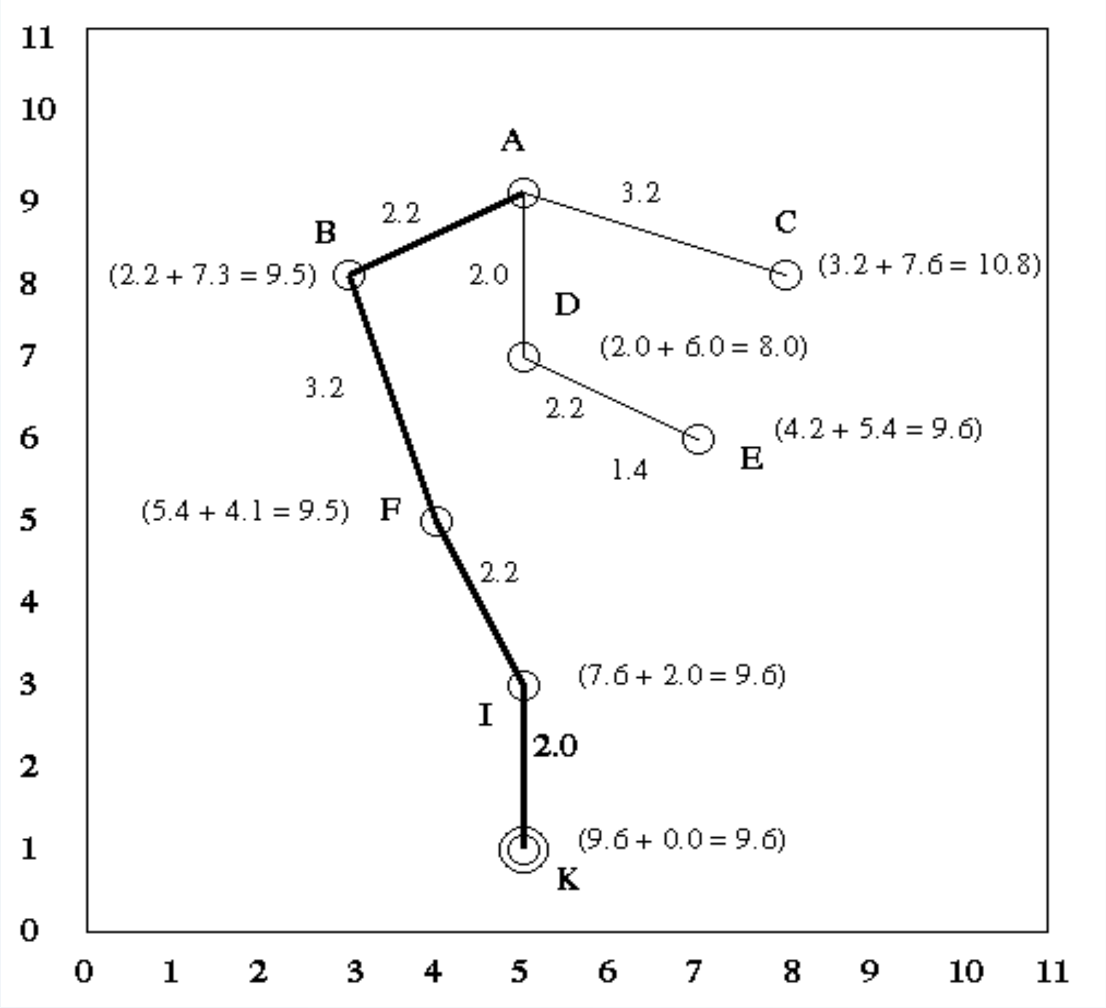

Greedy Best-First Exercise

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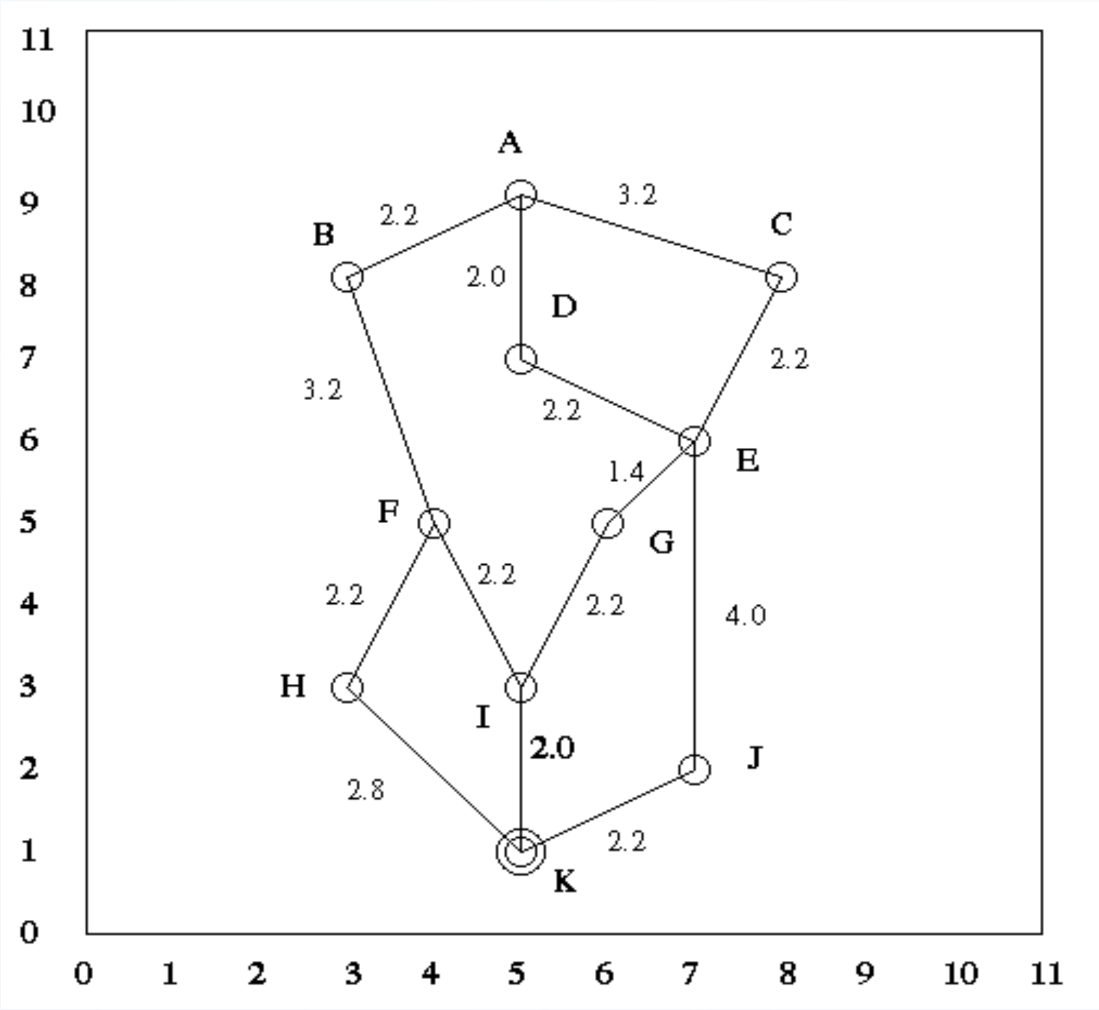

Solution to Greedy Best-First Exercise

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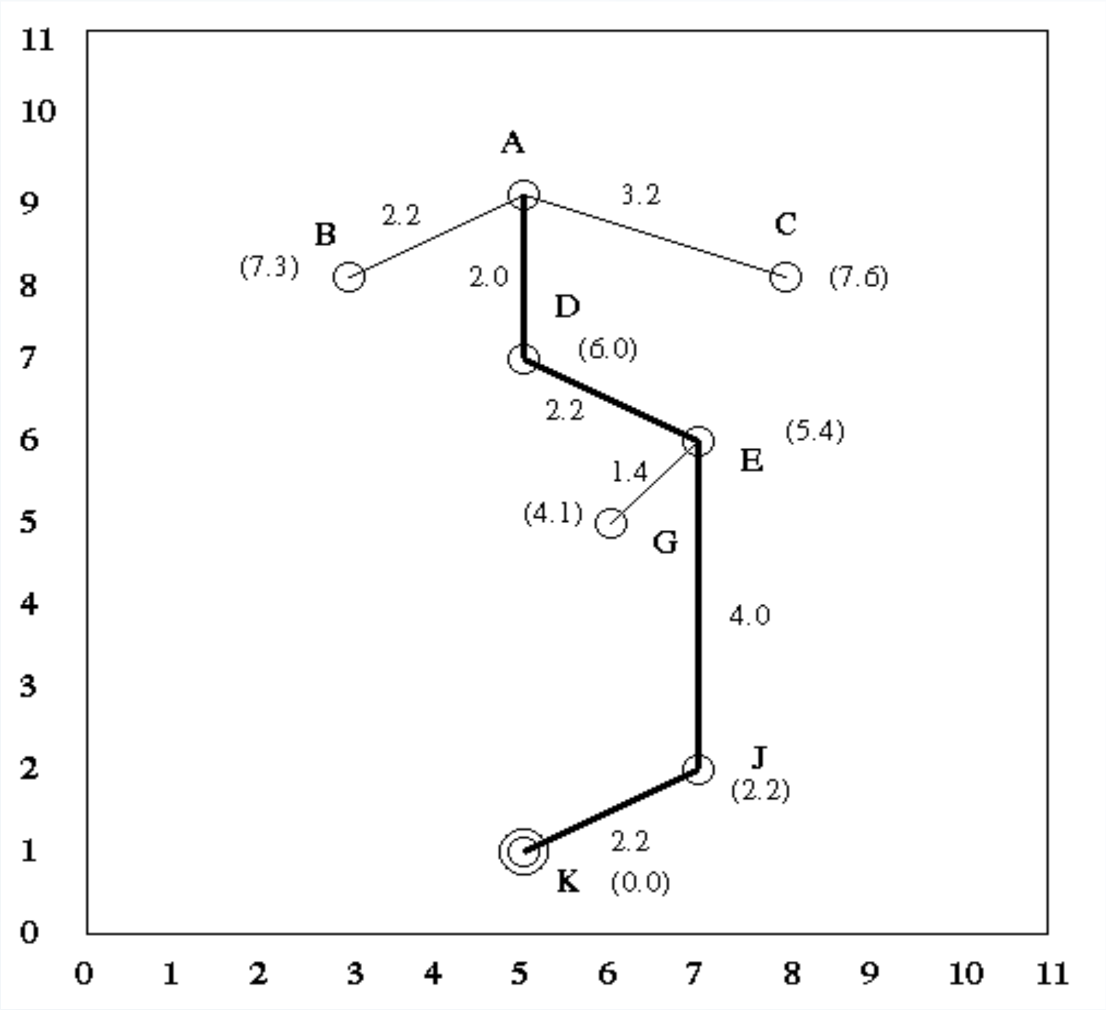
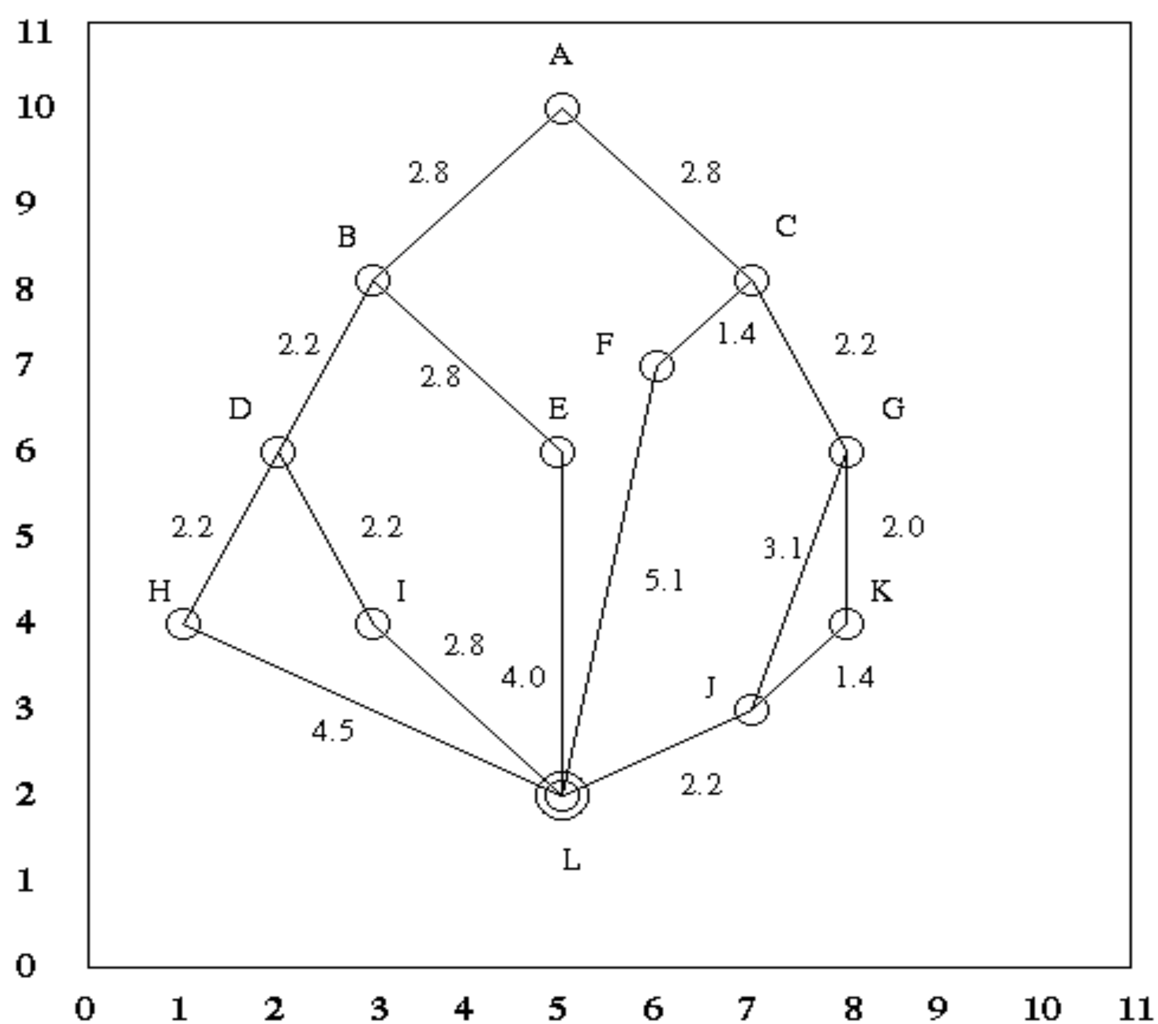


Another Exercise

Do 1) A* Search and 2) Greedy Best-Fit Search

Node	C	g(n)	h(n)
A	(5,10)	0.0	8.0
B	(3,8)	2.8	6.3
C	(7,8)	2.8	6.3
D	(2,6)	5.0	5.0
E	(5,6)	5.6	4.0
F	(6,7)	4.2	5.1
G	(8,6)	5.0	5.0
H	(1,4)	7.2	4.5
I	(3,4)	7.2	2.8
J	(7,3)	8.1	2.2
K	(8,4)	7.0	3.6
L	(5,2)	9.6	0.0



Admissible Heuristics

Based on [4]

A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic.

The heuristic function $h_{SLD}(n)$ *is* admissible because it never overestimates the actual road distance)

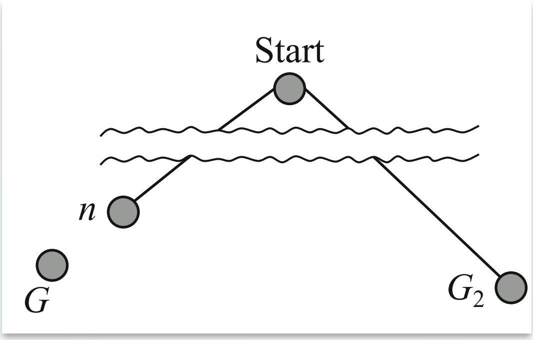
Theorem-1: If $h(n)$ is admissible, A^* using `TREE-SEARCH` is optimal.

Optimality of A^* (proof)

Based on [4]

Recall that $f(n) = g(n) + h(n)$

Now, suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



We want to prove:

$f(n) < f(G_2)$

(then A* will prefer n over G_2)

	$f(G_2) = g(G_2)$	since $h(G_2) = 0$
	$g(G_2) > g(G)$	since G_2 is suboptimal
	$f(G) = g(G)$	since $h(G) = 0$
Then	$f(G_2) > f(G)$	from above
	$h(n) \leq h^*(n)$	since h is admissible
	$g(n) + h(n) \leq g(n) + h^*(n)$	
Then	$f(n) \leq f(G)$	

Thus, A^* will never select G_2 for expansion

Optimality of A* (proof)

Recall that $f(n) = g(n) + h(n)$

Now, suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.

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$f(n) < f(G_2)$

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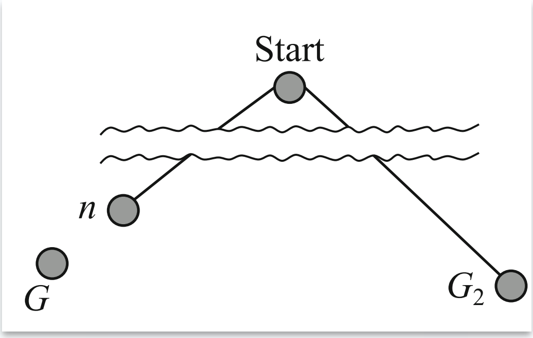


In other words:

$f(G_2) = g(G_2) + h(G_2) = g(G_2) > C^*$,

since G_2 is a goal on a non-optimal path (C^* is the optimal cost)

$f(n) = g(n) + h(n) \leq C^*$, since h is admissible

$f(n) \leq C^* < f(G_2)$, so G_2 will never be expanded

→ A* will not expand goals on sub-optimal paths

Properties of A*

- Complete: Yes

  unless there are infinitely many nodes with $f \leq f(G)$

- Time: Exponential

  because all nodes such that $f(n) \leq C^*$ are expanded!

- Space: Keeps all nodes in memory

  fringe is exponentially large

- Optimal: Yes

Memory Bounded Heuristic Search

How can we solve the memory problem for A* search?

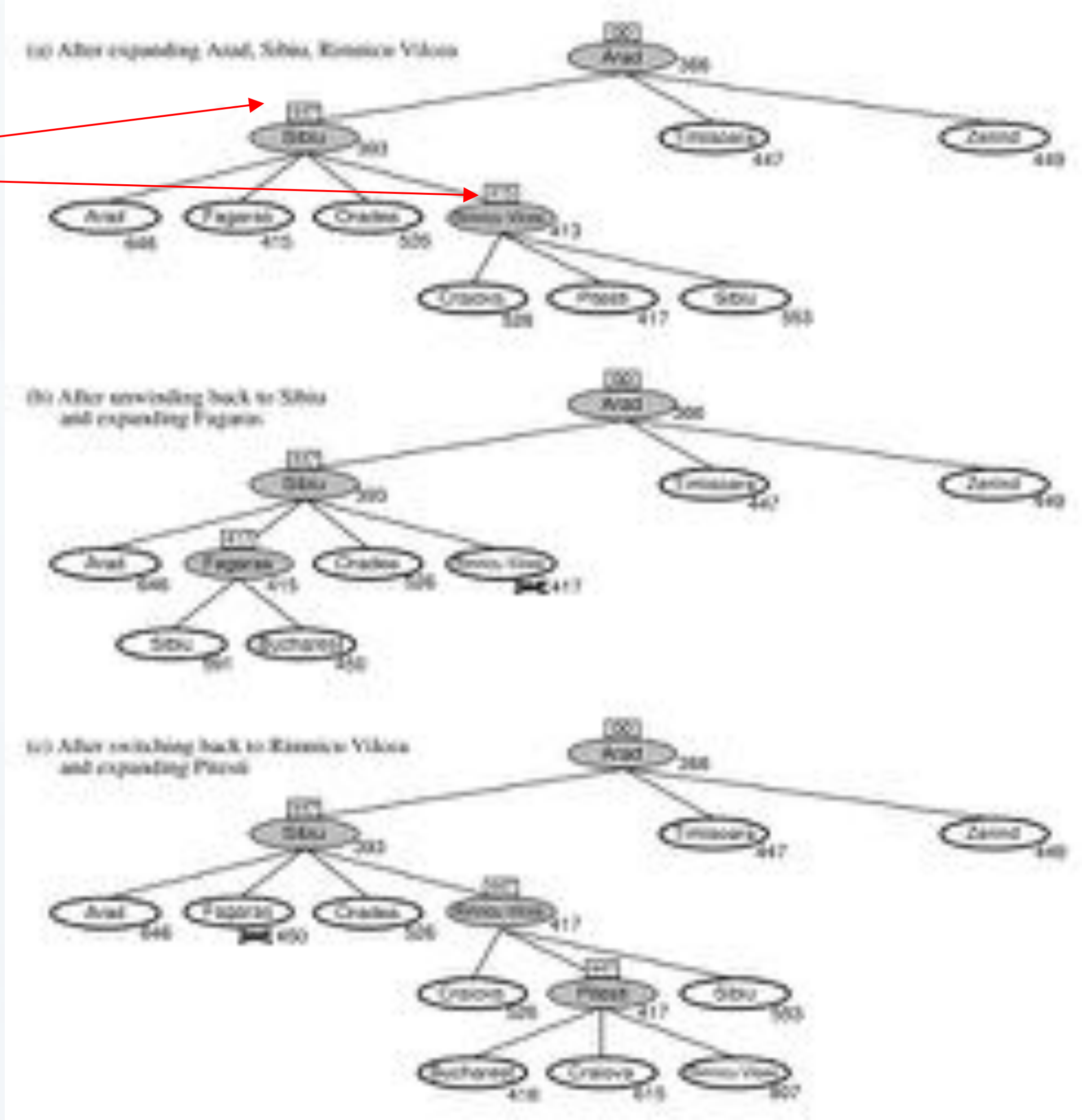
Idea: Try something like iterative deeping search, but the cutoff is f-cost (g+h) at each iteration, rather than depth first.

Two types of memory bounded heuristic searches:

- Recursive BFS
- SMA*

Recursive Best First Search (RBFS)

best alternative
over fringe nodes,
which are not children:
do I want to back up?



RBFS changes its mind very often in practice.

This is because the $f=g+h$ become more accurate (less optimistic) as we approach the goal. Hence, higher level nodes have smaller f-values and will be explored first.

Problem? If we have more memory we cannot make use of it.

Ay idea to improve this?

Simple Memory Bounded A* (SMA*)

- This is like A*, but when memory is full we delete the worst node (largest f-value).
- Like RBFS, we remember the best descendent in the branch we delete.
- If there is a tie (equal f-values) we first delete the oldest nodes first.
- SMA* finds the optimal *reachable* solution given the memory constraint.
- But time can still be exponential.

SMA* pseudocode

Based on [2]

```
function SMA*(problem) returns a solution sequence
  inputs: problem, a problem
  static: Queue, a queue of nodes ordered by f-cost

  Queue ← MAKE-QUEUE({MAKE-NODE(INITIAL-STATE[problem])})
  loop do
      if Queue is empty then return failure
      n ← deepest least-f-cost node in Queue
      if GOAL-TEST(n) then return success
      s ← NEXT-SUCCESSOR(n)
      if s is not a goal and is at maximum depth then
         f(s) ← ∞
      else
         f(s) ← MAX(f(n),g(s)+h(s))
      if all of n's successors have been generated then
         update n's f-cost and those of its ancestors if necessary
      if SUCCESSORS(n) all in memory then remove n from Queue
      if memory is full then
         delete shallowest, highest-f-cost node in Queue
         remove it from its parent's successor list
         insert its parent on Queue if necessary
      insert s in Queue
   end
```

Simple Memory-bounded A* (SMA*)

SMA* is a shortest path algorithm based on the A* algorithm.

The advantage of SMA* is that it uses a bounded memory, while the A* algorithm might need exponential memory.

All other characteristics of SMA* are inherited from A*.

How it works:

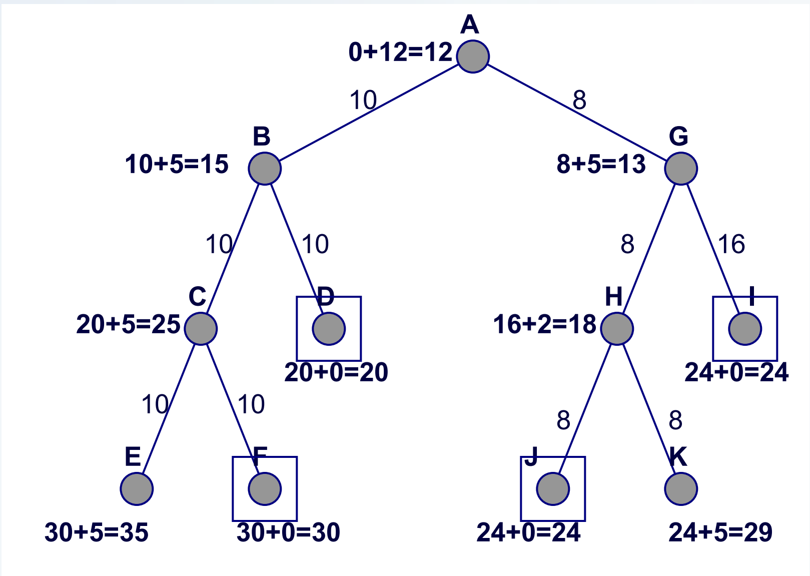
- Like A*, it expands the best leaf until memory is full.
- Drops the worst leaf node- the one with the highest f-value.
- Like RBFS, SMA* then backs up the value of the forgotten node to its parent.

Simple Memory-bounded A* (SMA*)

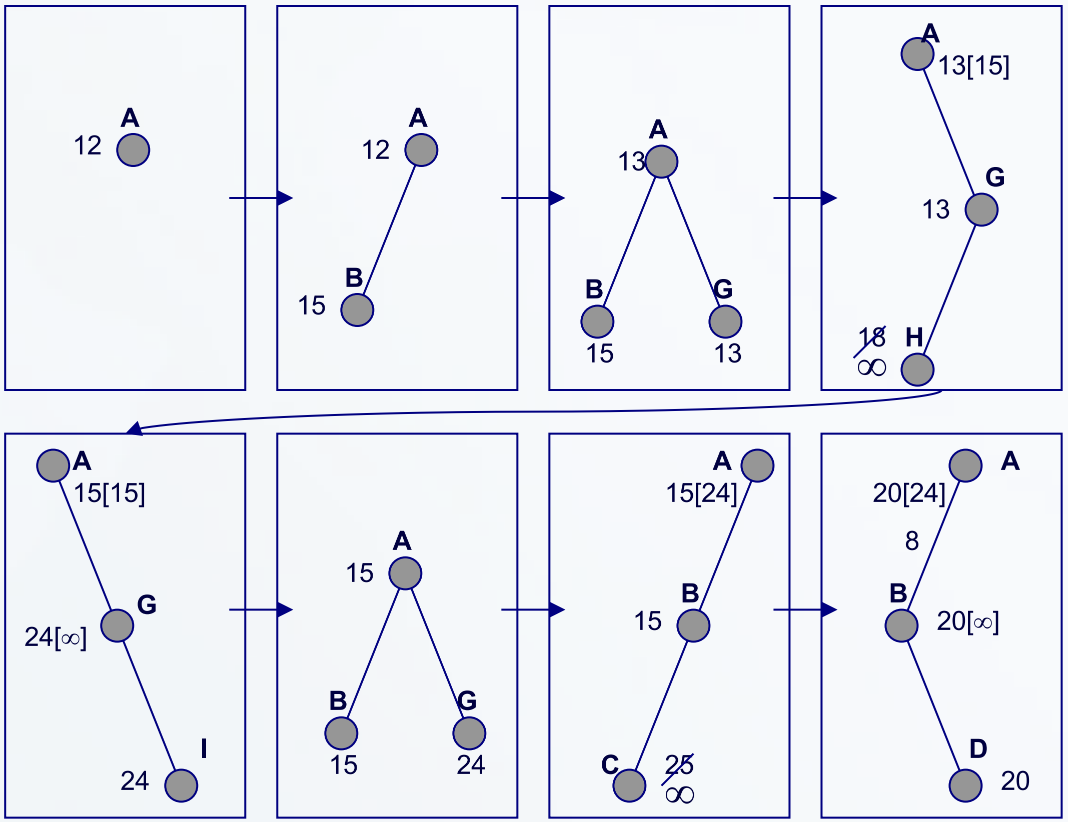
(Example with 3-node memory)

Search space

f = g+h   □ = goal



Progress of SMA*. Each node is labeled with its *current f*-cost. Values in parentheses show the value of the best forgotten descendant.



∞ is given to nodes that the path up to it uses all available memory.
Can tell when best solution found within memory constraint is optimal or not.

The Algorithm proceeds as follows [3]

1. At each stage, one successor is added to the deepest lowest-f-cost node that has some successors not currently in the tree. The left child B is added to the root A.
2. Now $f(A)$ is still 12, so we add the right child G ($f = 13$). Now that we have seen all the children of A, we can update its f-cost to the minimum of its children, that is, 13. The memory is now full.
3. G is now designated for expansion, but we must first drop a node to make room. We drop the shallowest highest-f-cost leaf, that is, B. When we have done this, we note that A's best forgotten descendant has $f = 15$, as shown in parentheses. We then add H, with $f(H) = 18$. Unfortunately, H is not a goal node, but the path to H uses up all the available memory. Hence, there is no way to find a solution through H, so we set $f(H) = \infty$.
4. G is expanded again. We drop H, and add I, with $f(I) = 24$. Now we have seen both successors of G, with values of ∞ and 24, so $f(G)$ becomes 24. $f(A)$ becomes 15, the minimum of 15 (forgotten successor value) and 24. Notice that I is a goal node, but it might not be the best solution because A's f-cost is only 15.
5. A is once again the most promising node, so B is generated for the second time. We have found that the path through G was not so great after all.
6. C, the first successor of B, is a nongoal node at the maximum depth, so $f(C) = \infty$.
7. To look at the second successor, D, we first drop C. Then $f(D) = 20$, and this value is inherited by B and A.
8. Now the deepest, lowest-f-cost node is D. D is therefore selected, and because it is a goal node, the search terminates.

SMA* Properties [2]

- It works with a heuristic, just as A*
- It is complete if the allowed memory is high enough to store the shallowest solution.
- It is optimal if the allowed memory is high enough to store the shallowest optimal solution, otherwise it will return the best solution that fits in the allowed memory.
- It avoids repeated states as long as the memory bound allows it
- It will use all memory available.
- Enlarging the memory bound of the algorithm will only speed up the calculation.
- When enough memory is available to contain the entire search tree, then calculation has an optimal speed

Admissible Heuristics

How can you invent a good admissible heuristic function?

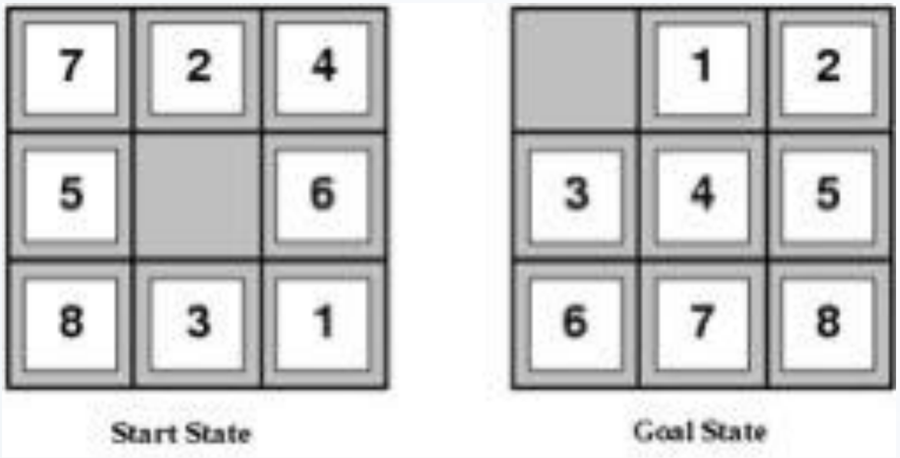
E.g., for the 8-puzzle

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance (i.e., no. of squares from desired location of each tile)



$h_1(S) = ?$

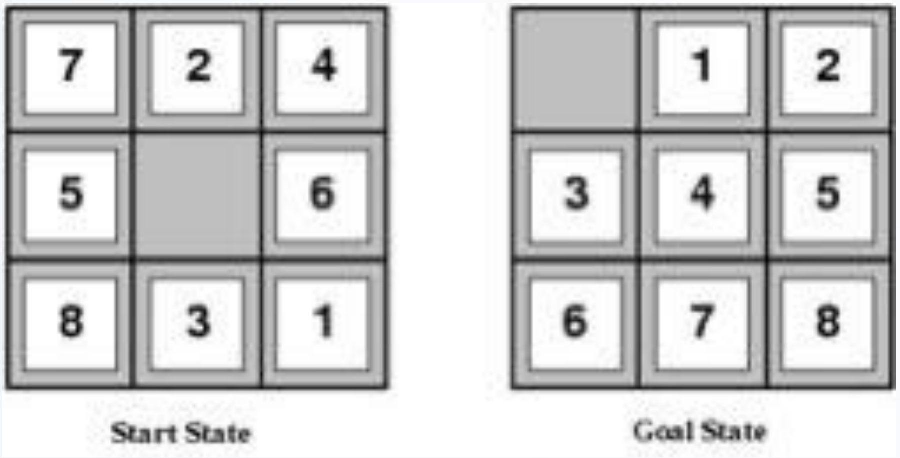
$h_2(S) = ?$

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance (i.e., no. of squares from desired location of each tile)



$h_1(S) = 8$

$h_2(S) = 3+1+2+2+2+3+3+2 = 18$

Dominance

If $h_2(n) \geq h_1(n)$ for all n, and both are admissible.

then h_2 dominates h_1

h_2 is better for search: it is guaranteed to expand less nodes.

Typical search costs (average number of nodes expanded):

d=12 IDS = 3,644,035 nodes
$A^*(h_1)$ = 227 nodes
$A^*(h_2)$ = 73 nodes

d=24 IDS = too many nodes
$A^*(h_1)$ = 39,135 nodes
$A^*(h_2)$ = 1,641 nodes

What to do If we have $h_1 \ldots h_m$, but none dominates the other?

➔ $h(n) = \max\{h_1(n), \ldots h_m(n)\}$

Relaxed Problems

A problem with fewer restrictions on the actions is called a relaxed problem.

The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.

If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution.

If the rules are relaxed so that a tile can move to any near square, then $h_2(n)$ gives the shortest solution.

Admissible Heuristics

How can you invent a good admissible heuristic function?

- ➢Try to relax the problem, from which an optimal solution can be found easily.
- ➢Learn from experience.

➔Can machines invite an admissible heuristic automatically?

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