Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2015

Set Theory

6.1. Basics of Set Theory

6.2 Properties of Sets and Element Argument

6.3 Algebraic Proofs

6.4 Boolean Algebras

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 6 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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6.3 Algebraic Proofs

In this lecture:

- **Part 1: Disapproving and Problem-Solving**
- Part 2: Algebraic Proofs of Sets

(Dis)proving

Prove that: For all sets A, B, and C, **(A - B) U (B - C)** ≠ ***A - C ?***

Example: All people except who are Palestinians with the set of Palestinians except who are female, are the same set as all people except who are female?

Counterexample 1: Let $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$, and $C = \{4, 5, 6, 7\}$. Then

$$A - B = \{1, 4\}, \quad B - C = \{2, 3\}, \quad \text{and} \quad A - C = \{1, 2\}.$$

Hence

$$(A - B) \cup (B - C) = \{1, 4\} \cup \{2, 3\} = \{1, 2, 3, 4\}, \quad \text{whereas} \quad A - C = \{1, 2\}.$$

Since $\{1, 2, 3, 4\} \neq \{1, 2\}$, we have that $(A - B) \cup (B - C) \neq A - C$.

Counterexample 2: Let $A = \emptyset$, $B = \{3\}$, and $C = \emptyset$. Then

$$A - B = \emptyset, \quad B - C = \{3\}, \quad \text{and} \quad A - C = \emptyset.$$

Hence $(A - B) \cup (B - C) = \emptyset \cup \{3\} = \{3\}$, whereas $A - C = \emptyset$.

Since $\{3\} \neq \emptyset$, we have that $(A - B) \cup (B - C) \neq A - C$.

Problem-Solving Strategy

How can you discover whether a given universal statement about sets is true or false?

حاول قليلا ان تثبت الصحة،
وان احسست عدم الصحة
حاول ايجاد مثال داحض،
ولكن ان احسست الصحة حاول الاثبات،
وهكذا...

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6.3 Algebraic Proofs

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Remember the following

$$\underbrace{A_1}_{A} \cap (\underbrace{A_2}_{B} \cup \underbrace{A_3}_{C}) = (\underbrace{A_1}_{A} \cap \underbrace{A_2}_{B}) \cup (\underbrace{A_1}_{A} \cap \underbrace{A_3}_{C}),$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\underbrace{(W \cap X)}_{A} \cap (\underbrace{Y}_{B} \cup \underbrace{Z}_{C}) = (\underbrace{(W \cap X)}_{A} \cap \underbrace{Y}_{B}) \cup (\underbrace{(W \cap X)}_{A} \cap \underbrace{Z}_{C}),$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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Algebraic Proofs

Deriving a Set Difference Property

Construct an algebraic proof that for all sets A, B, and C,

(A ∪ B) – C = (A – C) ∪ (B – C).

$(A \cup B) - C$	$= (A \cup B) \cap C^c$	by the set difference law
	$= C^c \cap (A \cup B)$	by the commutative law for ∩
	$= (C^c \cap A) \cup (C^c \cap B)$	by the distributive law
	$= (A \cap C^c) \cup (B \cap C^c)$	by the commutative law for ∩
	$= (A - C) \cup (B - C)$	by the set difference law.

Cite a property from Theorem 6.2.2 for every step of the proof.

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Algebraic Proofs

Deriving a Set Identity Using Properties of ∅

Construct an algebraic proof that for all sets A and B,

$$A - (A \cap B) = A - B.$$

$$\begin{aligned}
A - (A \cap B) &= A \cap (A \cap B)^c && \text{by the set difference law} \\
&= A \cap (A^c \cup B^c) && \text{by De Morgan's laws} \\
&= (A \cap A^c) \cup (A \cap B^c) && \text{by the distributive law} \\
&= \emptyset \cup (A \cap B^c) && \text{by the complement law} \\
&= (A \cap B^c) \cup \emptyset && \text{by the commutative law for } \cup \\
&= A \cap B^c && \text{by the identity law for } \cup \\
&= A - B && \text{by the set difference law.}
\end{aligned}$$

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